Combined Heat Conduction and Heat Radiation in One-Dimensional Solid

Toshiro Maruyama and Hiroaki Satake
Dept. of Chemical Engineering, Graduate School of Engineering, Kyoto University, Kyoto 615-8510, Japan

DOI 10.1002/aic.10671
Published online September 29, 2005 in Wiley InterScience (www.interscience.wiley.com).

The transient changes in the temperature of a solid material heated by radiation were obtained by numerical calculations using an unsteady one-dimensional (1-D) energy equation. The radiative loss of heat from the surface of a heated solid increases with an increase in the radiation flux. Therefore, the net blackbody radiation into a semi-infinite solid keeps a higher proportion of the initial flux at a lower dimensionless initial flux $I_0(0)/\alpha k(T_b - T_0)$. In the case of bidirectional radiation to a finite solid, the relatively homogeneous heating can be accomplished at the optical thickness $\alpha L \cong 10^0$. The heating rate is very low at $\alpha L \lesssim 10^{-1}$ resulting from the absorption of a small part of the radiation energy. On the other hand, the heating rate is high at $\alpha L \gtrsim 10^1$, but the heating is limited to a region near the wall, which results in an inhomogeneous temperature profile. At $\alpha L \gtrsim 10^2$, the temperature profiles become identical with that obtained for $\alpha L \to \infty$. © 2005 American Institute of Chemical Engineers *AIChE J,* 52: 478–483, 2006
Keywords: radiation, heat transfer, energy equation, optical thickness, blackbody

Introduction

Recently, radiative heating of materials has been applied in many fields of engineering and science. In particular, radiative heating using ultra-infrared light and microwaves has attracted increased attention with respect to the direct penetration of energy inside the solid, which results in a rapid and homogeneous heating of the materials. The present study discusses these unique characteristics of the radiation heat transfer in solids, on the basis of two simple 1-D models of transient heat transfer such as the heating of a semi-infinite solid by one-directional blackbody radiations and that of a finite solid by bidirectional radiations. In these models, the dominating dimensionless terms were derived by normalizing the basic equations of energy and radiation intensity. The transient temperature profiles were obtained by numerical calculations using the normalized equations with a finite-difference method. In these calculations, the physical properties of the solid were assumed to be independent of temperature. The results were compared with the solely conductive transfer from the surface, where the radiation energy is completely absorbed and changed into heat.

Correspondence concerning this article should be addressed to T. Maruyama at maruyama@cheme.kyoto-u.ac.jp.

Heating of Semi-Infinite Solid by One-Directional Radiation

First, we studied the radiation heat transfer in a solid of semi-infinite length, where the blackbody radiation is exchanged between two plain solids situated opposite each other. With keeping the temperature of the blackbody higher and constant as T_b, the transient changes in temperature of the one-dimensional heated solid of semi-infinite length were analyzed. A typical example for this model in the field of applications is the radiative heating using an ultra-infrared light.

Temperature Profile in Solid

Figure 1 shows the geometry of the blackbody of higher temperature T_b, which uniformly radiates energy in the x-direction. The heated solid of temperature T has semi-infinite length in the x-direction. The basic equation for the energy of the solid body is expressed as[1]

$$\rho C_p \partial T/\partial t = k\partial^2 T/\partial x^2 - \partial I/\partial x \qquad (1)$$

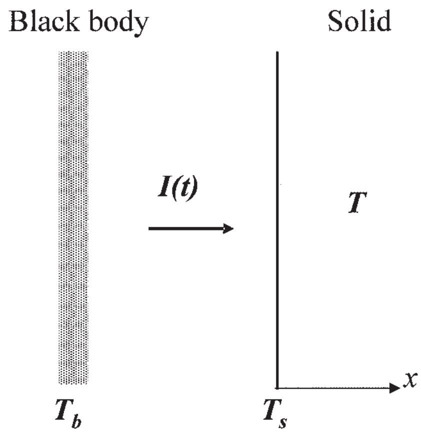

Figure 1. Geometry of the radiation and heated solid of semi-infinite length.

Initial Condition

$$T = T_0 \qquad \text{at } t = 0 \tag{2}$$

Boundary Condition

$$\partial T/\partial x = 0 \qquad \text{at } x = 0 \tag{3}$$

and

$$T = T_0 \qquad \text{at } x \rightarrow \infty \tag{4}$$

where ρ, C_p, and k are respectively the density, heat capacity, and heat conductivity of the solid and T_0 represents the constant temperature in the initial state. The second term in the right-hand side of Eq. 1 is the radiation energy absorbed in the solid. The profile of the radiation energy in the x-direction of the solid is expressed by the Beer–Lambert law:

$$I(t)/I_0(t) = \exp(-\alpha x) \tag{5}$$

where I_0 represents the intensity at the surface of the solid and α is the absorption coefficient. When the solid is approximated by a blackbody, the net energy taken by the solid is expressed as

$$I_0(t) = \sigma(T_b^4 - T_s^4) \tag{6}$$

where σ is the Stefan–Boltzmann constant (=5.67×10^{-8} J m^{-2} s^{-1} K^{-4}).[2]

Substitution of Eqs. 5 and 6 into Eq. 1 and further nondimensionalization yield the following equation:

$$\partial T^*/\partial\theta = \partial^2 T^*/\partial X^2 + [I_0(0)/\alpha k(T_b - T_0)] \times [I_o(\theta)/I_0(0)]\exp(-X) \tag{7}$$

Initial Condition

$$T^* = 0 \qquad \text{at } \theta = 0 \tag{8}$$

Boundary Condition

$$\partial T^*/\partial X = 0 \qquad \text{at } X = 0 \tag{9}$$

and

$$T^* = 0 \qquad \text{at } x \rightarrow \infty \tag{10}$$

where $\theta = k\alpha^2 t/\rho C_p$, $X = \alpha x$, and $T^* = (T - T_0)/(T_b - T_0)$. The dominating dimensionless term in Eq. 7 is $I_0(0)/\alpha k(T_b - T_0)$. The effect of the dimensionless initial flux $I_0(0)/\alpha k(T_b - T_0)$ on heat transfer is discussed below.

The transient temperature profiles were obtained from the numerical calculations using a finite-difference approximation[3] of Eq. 7 for different values of $I_0(0)/\alpha k(T_b - T_0)$ ranging from 10^{-1} to 10^2. The results obtained at $T_b = 800$ K and $T_0 = 300$ K are shown in Figure 2. A comparison of profiles in Figures 2a to 2d indicates that the profiles increase monotonically with increasing values of $I_0(0)/\alpha k(T_b - T_0)$. However, the profiles obtained for $I_0(0)/\alpha k(T_b - T_0)$ as 10 (Figure 2c) and 100 (Figure 2d) are identical and in good agreement. These results show that an increase in the dimensionless initial flux accelerates the increase in dimensionless temperature and hit the ceiling at $I_0(0)/\alpha k(T_b - T_0) \cong 10$ as a result of the attainment of the surface temperature to the upper limit value T_b.

The transient temperature profiles obtained for solely conductive heating in solid at $I_0(0)/\alpha k(T_b - T_0) = 0.1$ are shown in Figure 3. The profiles for conductive heating were obtained from the numerical calculations of the following equations:

$$\partial T^*/\partial\theta = \partial^2 T^*/\partial X^2 \tag{11}$$

Initial Condition

$$T^* = 0 \qquad \text{at } \theta = 0 \tag{12}$$

Boundary Condition

$$\partial T^*/\partial X = -I_0(\theta)/\alpha k(T_b - T_0) \qquad \text{at } X = 0 \tag{13}$$

and

$$T^* = 0 \qquad \text{at } x \rightarrow \infty \tag{14}$$

These equations express that the radiation is completely absorbed and changed to heat at the surface of the solid and the heat transfer in solid is solely conductive. The difference in profiles between Figure 2a and Figure 3 indicates the effects of the absorption of radiative energy in the inside of the solid. Accelerations in the temperature increase at $X \cong 1$ suggests that the radiation can directly heat the inside of a solid within the region of $X \lesssim 10$.

Net energy taken by the solid

The normalized net energy $I_o(\theta)/I_0(0)$ taken by the solid are shown in Figure 4 and its variation with the dimensionless time strongly depends on dimensionless initial flux $I_0(0)/\alpha k(T_b - T_0)$. In particular, $I_o(\theta)/I_0(0)$ rapidly decreases close to zero at $\theta \approx 1$ for the values of $I_0(0)/\alpha k(T_b - T_0) > 10$. This can be

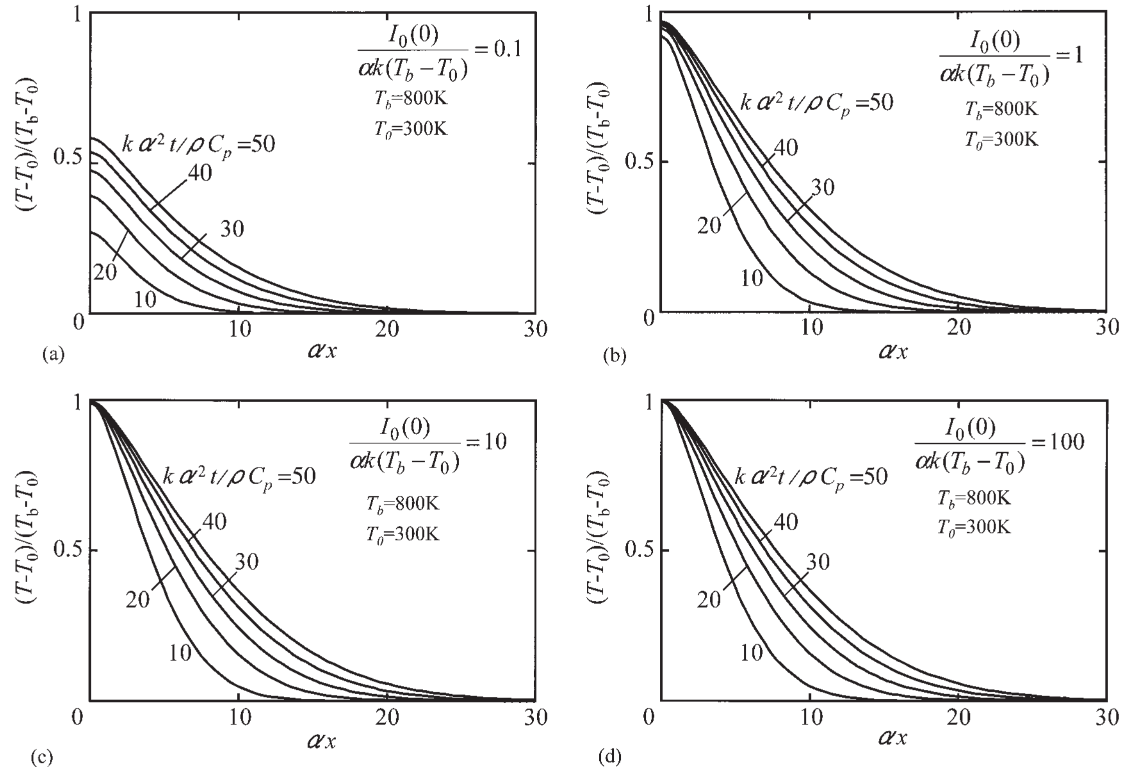

Figure 2. Transient temperature profiles of radiation heat transfer for the values of (a) $I_0(0)/\alpha k(T_b - T_0) = 10^{-1}$, (b) 10^0, (c) 10^1, and (d) 10^2.

attributed to a rapid increase in the surface temperature T_s of the solid at the large values of $I_0(0)/\alpha k(T_b - T_0)$. In the case of radiation heating, the energy directly penetrates into the solid at the optical distance αx of the order of 10^0. This results in a rapid increase in temperature within this optical thickness and consequently increases the radiative loss of heat from the surface of the solid. Therefore, the net blackbody radiation into a semi-infinite solid keeps a higher proportion of the initial flux at a lower dimensionless flux $I_0(0)/\alpha k(T_b - T_0)$.

Heating of Finite Solid by Bidirectional Radiation

Next, we studied 1-D heat transfer in a solid of finite length where the radiation is shed from two reflection surfaces situated opposite to the solid. In the present case, radiation from the solid surface is assumed to be negligibly small. A typical example for this model in the field of applications is the radiative heating using microwaves.

Profile of intensity of radiation in the solid

Figure 5 shows the geometry of the solid having a finite length of $2L$ and the reflection surfaces, which uniformly radiate energy in both the positive and negative directions of x. The intensity is kept constant as I_0 at the surface of the solid. The basic equation for the intensity of radiation in the solid is expressed as[4]

$$dI/dx = \pm \alpha I \tag{15}$$

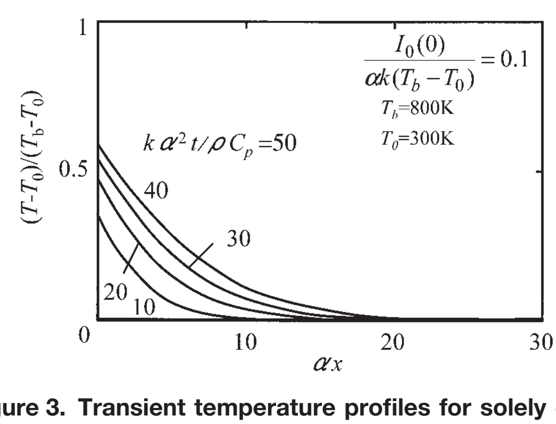

Figure 3. Transient temperature profiles for solely conductive heating in the solid at $I_0(0)/\alpha k(T_b - T_0) = 0.1$.

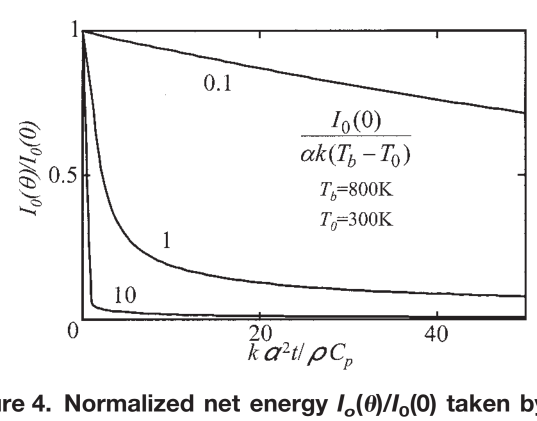

Figure 4. Normalized net energy $I_o(\theta)/I_0(0)$ taken by the solid as a function of θ.

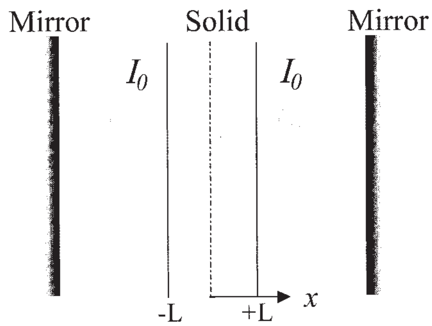

Figure 5. Geometry of the solid and reflection surfaces for bidirectional radiation.

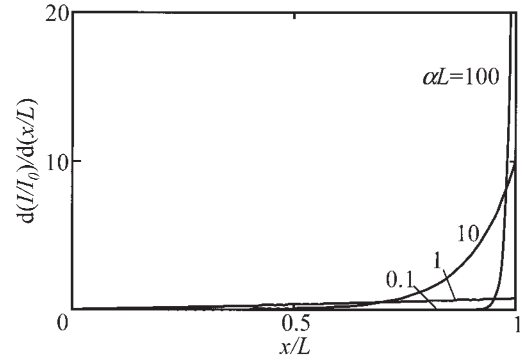

Figure 7. Profiles of absorbed radiation energy in the solid.

Boundary Condition

$$I = I_0 \qquad \text{at } x = \pm L \tag{16}$$

and

$$dI/dx = 0 \qquad \text{at } x = 0 \tag{17}$$

The profile of the intensity of radiation in the solid is expressed as

$$I/I_0 = \cosh(\alpha x)/\cosh(\alpha L) \tag{18}$$

Figure 6 shows the profiles of the intensity of radiation in the solid. A large difference in the profiles of radiation in the solid is observed depending on the optical thickness αL.

Temperature profile in the solid

The basic equation of energy is expressed as

$$\rho C_p \partial T/\partial t = k\partial^2 T/\partial x^2 + \partial I/\partial x \tag{19}$$

Initial Condition

$$T = T_0 \qquad \text{at } t = 0 \tag{20}$$

Boundary Condition

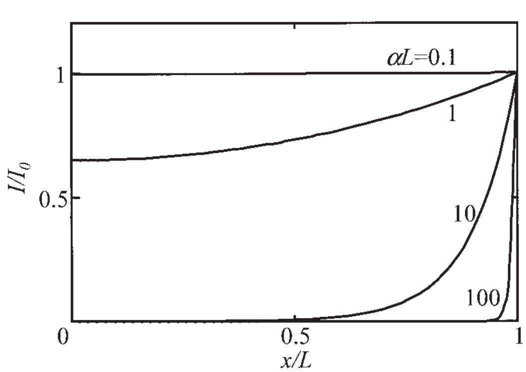

Figure 6. Profiles of intensity of radiation in the solid.

$$\partial T/\partial x = 0 \qquad \text{at } x = 0 \text{ and } L \tag{21}$$

The second term in the righthand side of Eq. 19 is the radiation energy absorbed in the solid. Figure 7 shows the profiles of the absorbed radiation energy in the solid. On increasing the optical thickness αL, the absorbed energy shows a rapid increase in a narrower region closer to the surface.

Substitution of Eq. 18 into Eq. 19 and further nondimensionalization yield the following equation:

$$\partial T^*/\partial \theta = \partial^2 T^*/\partial X^2 + (\alpha L^2 I_0/kT_0)\sinh(\alpha L X)/\cosh(\alpha L) \tag{22}$$

Initial Condition

$$T^* = 1 \qquad \text{at } \theta = 0 \tag{23}$$

Boundary Condition

$$\partial T^*/\partial X = 0 \qquad \text{at } X = 0, 1 \tag{24}$$

where $\theta = kt/\rho C_p L^2$, $X = x/L$, and $T^* = T/T_0$.

In Eq. 22, LI_0/kT_0 and αL (optical thickness) are two dimensionless terms. The effects of LI_0/kT_0 are linear and those of αL are nonlinear to the dimensional temperature. The effects of optical thickness αL are discussed below.

The transient temperature profiles were obtained using numerical calculations of a finite-difference approximation of Eq. 22 for different values of αL ranging from 10^{-1} to 10^2. The results obtained at $LI_0/kT_0 = 10$ are shown in Figure 8. For comparison, the transient temperature profiles for $LI_0/kT_0 = 10$ at $\alpha L \to \infty$ are shown in Figure 9. The profiles were obtained from the numerical calculations of the following equations:

$$\partial T^*/\partial \theta = \partial^2 T^*/\partial X^2 \tag{25}$$

Initial Condition

$$T^* = 1 \qquad \text{at } \theta = 0 \tag{26}$$

Boundary Condition

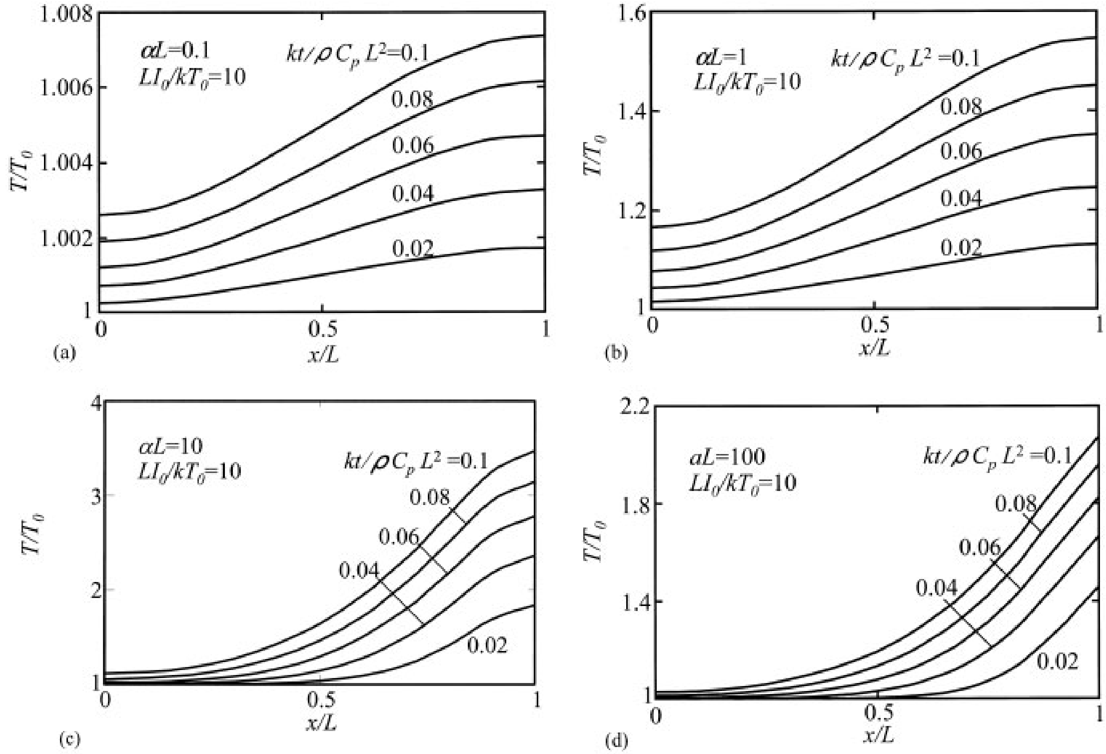

Figure 8. Transient temperature profiles at $LI_0/kT_0 = 10$ for the values of (a) $\alpha L = 10^{-1}$, (b) 10^0, (c) 10^1, and (d) 10^2.

$$\partial T^*/\partial X = LI_0/kT_0 \qquad \text{at } X = 1 \tag{27}$$

and

$$\partial T^*/\partial X = 0 \qquad \text{at } X = 0 \tag{28}$$

These equations express that the radiation is absorbed and changed to heat at the surface and the heat transfer in the solid is solely conductive.

Apparently, the results in Figure 9 are in good agreement with the results obtained at $\alpha L > 100$ (Figure 8d) and this suggests that the heat transfer in the solid is solely conductive at $\alpha L > 100$. On the other hand, the results presented in Figures 8a and 8b show the other type of temperature profile, revealing a relatively small temperature decrease from the surface to the center and the same order of magnitude of temperatures throughout the cross section. These results reveal that the condition of $\alpha L \lesssim 1$ is preferable to heat the material homogeneously. At $\alpha L \lesssim 10^{-1}$, a very small part of the radiation energy can be absorbed by the solid as shown in Figure 7, which results in very low heating rate. On the other hand, the heating rate is high at $\alpha L \gtrsim 10^1$ as shown in Figure 7. However, all the radiation energy can be absorbed by the solid within a limited region near the wall, which results in an inhomogeneous heating. At $\alpha L \gtrsim 10^2$, the change in temperature profile shows the limiting one at $\alpha L \rightarrow \infty$, which reveals a sole conducting heat transfer in the solid with the most inhomogeneous heating.

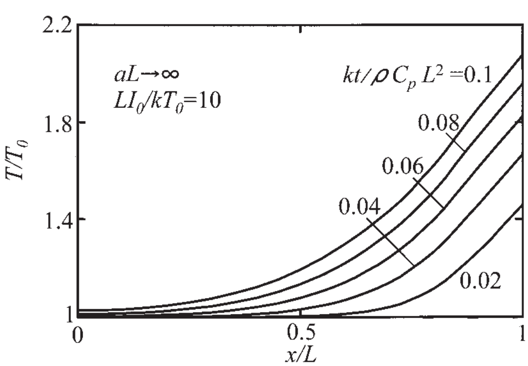

Figure 9. Transient temperature profiles for $LI_0/kT_0 = 10$ at $\alpha L \rightarrow \infty$.

Finally, the effects of the radiation from a solid surface are briefly described. The radiation from the solid surface is first assumed to be negligibly small in this analysis. However, the intensity of the net radiation is not kept constant at the surface of the solid, when the solid is approximated to be a blackbody. Because the radiation loss from the surface of the solid has a fourth-power dependency on the surface temperature, a decrease in the rate of temperature increase is expected on increasing surface temperature. Further, the influence of this effect is expected to be substantial at larger values of LI_0/kT_0 and αL. However, the results obtained in the present study about the homogeneous heating are consistently applicable because the temperature takes similar profiles in the x-direction with a decrease only in the magnitude.

Conclusions

In radiation heating, the energy directly penetrates into solid at the optical distance αx of the order of 10^0, which promotes a rapid increase in temperature. This results in an increase in the radiative loss of heat from the surface of the solid with an increase in the radiation flux. Thus, the net blackbody radiation into a semi-infinite solid keeps a higher proportion of the initial flux for lower dimensionless initial flux $I_0(0)/\alpha k(T_b - T_0)$. In the case of bidirectional radiation to a finite solid, a homogeneous heating is influenced by the optical thickness αL of the solid. At the optical thickness $\alpha L \cong 10^0$, the relatively homogeneous heating can be accomplished. At $\alpha L \lesssim 10^{-1}$, a very small part of the radiation energy is absorbed by the solid and this results in very low heating rate. On the other hand, all the radiation energy is absorbed by the solid at $\alpha L \gtrsim 10^1$ and this results in a high heating rate. However, the heating is limited to the region near the wall, which leads to an inhomogeneous temperature distribution. At $\alpha L \gtrsim 10^2$, the temperature profiles become identical with those obtained for $\alpha L \rightarrow \infty$.

Acknowledgments

The authors thank Dr. B. Balamurugan for his contribution to significant improvements of the manuscript.

Literature Cited

1. Ismail KAR, Henriquez JR. Modeling and simulation of a simple glass window. *Sol Energy Mater Sol Cells.* 2003;80:355-374.
2. Siegel R, Howell JR. *Thermal Radiation Heat Transfer*. New York, NY: McGraw-Hill; 1972.
3. Yamasaki K. *Introduction of Numeric Method of Partial Differential Equation*. 1st Edition. Tokyo, Japan: Morikita Publishing; 1993.
4. Cassano AE, Silveston PL, Smith JM. Photochemical reaction engineering. *Ind Eng Chem.* 1967;59:18-38.

Manuscript received May 7, 2005, and revision received July 22, 2005.